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SHORT COMMENTRY

# Avoiding Two Major Problems Associated with Statistical Tests: One-way Analysis of Variance

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ABSTRACT

The two major problems with one-way analysis of variance (ANOVA) are (1) the false assumption of having independent normal distributions and (2) the induced non-metric analysis space that differs from the space of the observed data being analyzed. The multi-response permutation procedure (MRPP) test is introduced because it is a generalized statistic that provides descriptions of both problems and their solutions [1-3]. First, permutation statistical methods are entirely data dependent, i.e., are dependent only on the observed data in question and, second, permutation methods avoid fabricated assumptions such as the data being randomly selected from independent normal distributions. MRPP also provides a simple description of the analysis spaces in question.

## Multi-response Permutation Procedures

Let $\Omega = \{\omega_1, \ldots, \omega_N\}$ denoteafinite collection (sample) of objects obtained from a target population of interest. Let $x_I' = (x_{1I}, \ldots, x_{rI})$ denoter response measurements on object $\omega_I \left(I = 1, \ldots, N\right)$, and let $S_1, \ldots, S_{g+1}$ designate an exhaustive partitioning of the N objects comprising $\Omega$ into $g+1$ disjoint groups. Also let $\Delta_{I,J}$ be a symmetric distance function value of the response measurements associated with the objects $\omega_I$ and $\omega_J$. The statistic underlying MRPP is given by

$$\delta = \sum_{i=1}^{g} C_i \xi_i \ ,$$

where

$$\xi_i = \binom{n_i}{2}^{-1} \sum_{I<J} \Delta_{I,J} \Psi(\omega_I)$$

is the average distance function value for all distinct pairs of objects in group $S_i (i = 1, \ldots, g)$, $n_i \geq 2$ is the number of objects classified into group $S_i$ $(i = 1, \ldots, g)$, $K = \sum_{i=1}^{g} n_i$, $n_{g+1} = N - K \geq 0$ is the number of remaining (unclassified) objects in the excess group $S_{g+1}$ (this is an empty subgroup in many applications), $\sum_{I<J}$ is the sum over all *I* and *J* such that $1 \leq I < J \leq N$, $\Psi(\omega_I) = 1$ if $\omega_I$ belongs to $S_i$ and 0 otherwise, $C_i > 0$ ( $i = 1, \ldots, g$ ), and $\sum_{i=1}^{g} C_i = 1$. The underlying permutation distribution of $\delta$ (the null hypothesis) assigns equal probabilities to the

$$M = \frac{N!}{\prod_{i=1}^{g+1} n_i!}$$

possible allocations of the *N* objects to the $g+1$ subgroups. The mean, variance, and skewness of $\delta$ under the null hypothesis are denoted by $\mu_\delta$, $\sigma_\delta^2$, and $\gamma_\delta$, respectively. Under the null hypothesis, preliminary findings indicated some situations when the asymptotic distribution of $N(\delta - \mu_\delta)$ is nondegenerate with $\gamma_\delta$ being substantially negative [4-6]. Based on results due to Sen [7,8], O'Reilly and Mielke [9] presented general theorems for the multivariate case of MRPP which characterize situations in which the distribution of $N^{1/2}(\delta - \mu_\delta)$ is asymptotically normal under the null hypothesis. Brockwell et al., [10] presented theorems for the univariate case of MRPP which delineate distributions for situations (probably the most important situations) in which the nondegenerate distribution of $N(\delta - \mu_\delta)$ is *not* asymptotically normal under the null hypothesis (with rare exceptions, invariance principles fail for these situations). The multivariate generalizations of the results by Brockwell et al., [10] and special situations analogous to those considered by Mielke and Sen [11] for linear rank-order statistics are open questions which require further attention.

The symmetric distance function, $\Delta_{I,J}$, is extremely important since it defines the structure of the underlying analysis space of MRPP. The form of the symmetric distance functions considered in this paper is confined to

$$\Delta_{I,J} = \left( \sum_{k=1}^{r} \left| x_{kI} - x_{kJ} \right|^p \right)^{v/p},$$

where $p \ge 1$ and $v > 0$ (pis not relevant when $r = 1$). In particular, the underlying analysis space of MRPP is nonmetric when $v > 1$ (i.e., the triangle inequality property of a metric space fails) and is metric when $v \le 1$ (a distorted metric space when $v < 1$). The analysis space of MRPP is a Euclidean space when $p = 2$ and $v = 1$. While the validity of a permutation test is not affected by these geometric considerations, the rejection region of any test is highly dependent on the underlying geometry. This is a geometry problem (i.e., either a nonmetric or a distorted metric space) that will affect the power of a permutation test. The results of a permutation statistical test will surely be misleading if the rejection region is incomprehensible. As a consequence, the choiceof $p = 2$ and $v = 1$ is recommended for routine applications. This may be acontroversial conclusion since, as subsequently demonstrated, the majority of permutation tests presently used in routine applications are based on $v = 2$.

### 5. Relation to Well-known Methods

The relation between the permutation version of one-way analysis of variance (two-sample *t* test when $g = 2$) and MRPP is described. Let

$$F = \frac{MS_A}{MS_W}$$

be the ordinary one-way analysis of variance statistic. If $g \ge 2$, $v = 2$, $r = 1$,

$$A = \sum_{I=1}^{N} x_{1I}, \quad B = \sum_{I=1}^{N} x_{1I}^2, \quad N = K, \quad \text{and} \quad C_i = \frac{n_i - 1}{N - g}$$

for $i = 1, \ldots, g$, then the identity relating *F* and $\delta$ is given by

$$N\delta = \frac{2(NB - A^2)}{N - g + (g-1)F}.$$

Because *F* is based on $v = 2$, the previously mentioned geometry problem of the underlying analysis space is a relevant concern for the permutation version of one-way analysis of variance.

Consider the *F* statistic under the usual normality assumption where the value of the *I* thof *g* groups is designated as *w* and is allowed to vary, while the remaining $N-1$ values remain fixed. When *w* becomes either very large or very small relative to the $N-1$ fixed values, the value of *F* approaches

$$\frac{(N-g)(N-n_i)}{(g-1)N(n_i - 1)}.$$

If $n_i = N/g$, then the value of *F* is 1. This result explains the over whelming influence of a single value relative to a fixed set of values for the *F* test. On the other hand, with $v = 1$, the permutation probability value is relatively unaffected by the presence of a single extreme value. To demonstrate that permutation tests with $v = 1$ are robust, consider the data given in Table 1 for comparing two groups of sizes $n_1 = n_2 = 13$. While $N - 1 = 25$ of the $N = 26$ values are fixed, one value of Group1, designated by *w* is allowed to vary in order to determine its effect on the exact two-sided probability values. Extreme examples of *w* and their associated probability values are given in Table2. The three sets of probability values given in Table 2 associated with each value of *w* correspond to the permutation tests with $v = 1$ and $v = 2$, and with the two-sample *t* test, respectively.

**Table1:** Frequencies of observed values for Groups 1 and 2 with *N*=26 subjects assigned to *g*=2 groups with *n*=13 subjects in each group.

| *Value* | *Group 1* | *Group 2* |
|---|---|---|
| 445.6 | 1 | 0 |
| 445.7 | 2 | 0 |
| 445.8 | 4 | 0 |
| 445.9 | 4 | 1 |
| 446.0 | 1 | 3 |
| 446.1 | 0 | 4 |
| 446.2 | 0 | 3 |
| 446.3 | 0 | 2 |
| *w* | 1 | 0 |

The exact permutation probability values with $v = 1$ for low and high values of *w*, relative to the 25 fixed values, are $4.04\times10^{-6}$ and $9.81\times10^{-5}$, respectively. The probability values are stable, consistent, and are not affected by the extreme magnitudes of *w* in either direction. In contrast, the permutation test probability values with $v = 2$ are $4.04\times10^{-6}$ for small values of *w* and 1.000 for large values of *w*, relative to the fixed values. Finally, the two-sample *t* test probability values approach a common probability value (i.e.,*P*=0.327) as *w* becomes either very small or very large, relative to the 25 fixed values. Due to the extreme values of *w*, the two-sample *t* test is unable to detect the obvious difference in location between Groups 1 and 2. The results of Table 2 imply that Euclidean distance-based permutation methods are so robustto extreme values that they never need the data manipulation techniques devised for squared Euclidean distance-based permutation methods, such as trun cations and rank-order statistic transformations.

**Table 2:** Frequencies of observed values for groups 1 and 2 with *N*=26 subjects assigned to *g*=2 groups with *n*=13 subjects in each group.

| *w* | *v = 1* | *v = 2* | *t test* |
|---|---|---|---|
| 107.7 | $4.04\times10^{-6}$ | $4.04\times10^{-6}$ | 0.322 |
| 437.7 | $4.04\times10^{-6}$ | $4.04\times10^{-6}$ | 0.153 |
| 443.7 | $4.04\times10^{-6}$ | $4.04\times10^{-6}$ | $1.16\times10^{-2}$ |
| 444.7 | $4.04\times10^{-6}$ | $4.04\times10^{-6}$ | $5.69\times10^{-4}$ |
| 445.7 | $4.04\times10^{-6}$ | $4.04\times10^{-6}$ | $5.84\times10^{-7}$ |
| 446.7 | $9.81\times10^{-5}$ | $9.82\times10^{-3}$ | $9.16\times10^{-3}$ |
| 447.7 | $9.81\times10^{-5}$ | 0.430 | 0.320 |
| 449.7 | $9.81\times10^{-5}$ | 1.000 | 1.000 |
| 453.7 | $9.81\times10^{-5}$ | 1.000 | 0.617 |
| 783.7 | $9.81\times10^{-5}$ | 1.000 | 0.333 |

## 6. Summary

The major implication of this paper is that all environmental research should be based on $v = 1$ (a Euclidean analysis space) rather than $v = 2$ (a squared Euclidean analysis space). Specifically, $v = 1$ yields statistical tests with congruent data and analysis spaces. Also, the data-dependent permutation approach introduced by R.A. Fisher [12] and E.J.G. Pitman [13] eliminates the analytic guessing game of an underlying distribution (such as normality). An investigator in most any field of study should not be concerned about a statistical analysis being overwhelmed by a single value as presently occurs with statistical tests based on $v = 2$.

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